Application of Structural Fire Design, 19-20 April 2013, Prague, Czech Republic

ANALYSIS OF CURVED REINFORCED CONCRETE BEAM IN FIRE CONDITIONS

Dušan Ružić [a,b], Miran Saje [b], Igor Planinc [b], Tomaž Hozjan [b]

[a] ELEA iC d.o.o. Civil engineering and consulting, Ljubljana, Slovenia
[b] University of Ljubljana, Faculty of Civil and Geodetic Engineering, Ljubljana, Slovenia

Abstract

In the paper a novel strain-based finite-element numerical model for the fire analysis of curved reinforced concrete (RC) beams is presented. In addition, the effect of load level and boundary conditions on fire resistance of the curved RC beam is observed.

Keywords: curved RC beam, fire, FEM, Reissner beam, moisture transfer, heat transfer, high temperatures

INTRODUCTION

To design and build significant/important engineering structures properly, such as tunnels, bridges, power plants and dams, certain requirements concerning mechanical resistance and stability of structures must be fulfilled. By fulfilling those requirements we assure engineering structures have sufficient level of safety. Insufficient level of safety can lead into human loss, material damage and pollution of environment. One of the requirements concerning mechanical resistance and stability of structures is fire safety.

Concrete is a heterogeneous material consisting of solid matrix, water and gaseous mixture of dry air and water vapour. When exposed to high temperatures (e.g. in fire) physical and chemical processes within concrete structure occur, such as heat transfer as a result of conduction and convection, release of chemically bonded water and evaporation of free water. This results in lower load bearing capability and higher deformability of concrete structure.

In this paper we present a new numerical model for fire analysis of curved RC beams. Few models for curved beams exposed to fire can be found in the literature, mostly for composite steel-concrete arches (Heidarpour et al., 2010) and steel arches (Bradford, 2006) exposed to thermal loading are presented. The new numerical model consists of three consecutive mathematically uncoupled phases. In the first phase we describe the time dependant change of temperatures of the fire compartment surrounding the beam. In the second (hygro-thermal) phase, heat flux affecting the surface of the beam due to convective and radiative heat flows from the surrounding fire compartment is accounted for in boundary conditions and temperature, pore pressure and free water content in a characteristic cross-section of the curved RC beam are determined implementing the model of Davie (Davie et al., 2006). In the third phase of the analysis the mechanical behaviour (stress-strain state evolution) of the beam during fire employs the geometrically non-linear theory of Reissner (Reissner, 1972) and accounts for axial, shear and flexural deformations. The system of governing equations of the model is solved numerically using the modified principle of virtual work and a strain-based Galerkin-type of FEM. The non-linear stress-strain relations for concrete and rebars at elevated temperatures and the rules for the reduction of material parameters due to an increased temperature are taken from European building code EC2 (EC2, 2004).

1 NUMERICAL MODEL

The model for fire analysis of curved RC beams, proposed in this paper, consists of three mathematically uncoupled phases. In the second and the third phase of the fire analysis the time of fire is divided into time intervals [t^{i-1}, t^i], where unknowns are determined with the Newton’s iteration method at each time t^i. In what follows a brief presentation of the three phases will be given.

1.1 Fire curves

Time dependent change of temperatures in a fire compartment depends on many parameters and is therefore difficult to predict. Engineers avoid such problem by using simplified parametric temperature-time curves, which define the relation between a gas temperature in the fire compartment and time for standardized situations. One of these fire curves is the hydrocarbon fire curve of the European building code EC1 (EC1, 2004), which represents a fully developed tunnel fire. Once the gas temperature in the fire compartment has been defined, heat and mass transfer in the curved beam can be analysed.

1.2 Heat and mass transport model

A coupled heat and moisture transfer in concrete exposed to fire is described by three governing equations of mass conservation of free water, water vapour and dry air and by a governing equation of energy conservation (Davie et al., 2006):

Water conservation:
$$\frac{\partial\left(\overline{\rho_{FW}}\right)}{\partial t} = -\nabla \mathbf{J}_{FW} - \dot{E}_{FW} + \frac{\partial\left(\overline{\rho_{D}}\right)}{\partial t} \tag{1}$$

Water vapour conservation:
$$\frac{\partial\left(\overline{\rho_{V}}\right)}{\partial t} = -\nabla \mathbf{J}_{V} + \dot{E}_{FW} \tag{2}$$

Air conservation:
$$\frac{\partial\left(\overline{\rho_{A}}\right)}{\partial t} = -\nabla \mathbf{J}_{A} \tag{3}$$

Energy conservation:
$$\left(\underline{\rho C}\right)\frac{\partial T}{\partial t} = -\nabla \cdot \left(-k\nabla T\right) - \left(\underline{\rho C \mathbf{v}}\right)\cdot \nabla T - \lambda_E \dot{E}_{FW} - \lambda_D \frac{\partial\left(\overline{\rho_{D}}\right)}{\partial t} \tag{4}$$

In *Eqs. (1)–(4)* $\bar{\rho}_i$ is the density (mass concentration) of a phase i, $\mathbf{J}_i$ is the mass flux of each phase i per unit volume of gaseous material, $\dot{E}_{FW}$ is the rate of evaporation of free water (including desorption), and t is time. Index i represents phases of concrete, FW is free water, V is water vapour and A is dry air. In *Eq. (4)* $\underline{\rho C}$ is heat capacity of concrete, k is thermal conductivity of concrete, $\underline{\rho C \mathbf{v}}$ relates to the energy transferred by fluid flow, λ_E is the specific heat of evaporation, λ_D is specific heat of dehydration and T is the absolute temperature.

By summing *Eq. (1)* and *(2)* we obtain three partial differential equations. The solution is obtained numerically with the finite element method, where the primary unknowns of the problem are temperature T, pressure of gaseous mixture of water vapour and dry air P_G and water vapour content $\overline{\rho_V}$. For a detailed description of the problem and its numerical formulation, see Davie et al. (2006) or Kolšek (2013) for application to fire analysis of composite structures.

1.3 Mechanical model

Once the time and space evolution of temperature and pore pressure in the beam have been obtained, the third phase of the fire analysis can be performed, where the stress-strain state of the beam during fire is finally determined.

In the mechanical model the curved RC beam is modelled by the kinematically exact planar beam model of Reissner (Reissner, 1972). It is assumed that the compatibility of deformations at the contact of the reinforcement and concrete holds. Furthermore, the beam element is assumed to be under a time-dependant temperature loading and conservative distributed forces p_X, p_Z and m_Y. The related governing equations are:

Kinematic equations:
$$x' + u' - (1+\varepsilon)\cos\varphi - \gamma \sin\varphi = 0 \tag{5}$$

$$z' + w' + (1+\varepsilon)\sin\varphi - \gamma \cos\varphi = 0 \tag{6}$$

$$\varphi' - \kappa_0 - \kappa = 0 \tag{7}$$

Equilibrium equations:
$$R_X' + p_X = 0 \tag{8}$$

$$R_Z' + p_Z = 0 \tag{9}$$

$$M' - (1+\varepsilon)\, Q + \gamma\, N + m_Y = 0 \tag{10}$$

$$N = R_X \cos\varphi - R_Z \sin\varphi \tag{11}$$

$$Q = R_X \sin\varphi + R_Z \cos\varphi \tag{12}$$

Constitutive equations:
$$N - N_c = 0 \tag{13}$$

$$Q - Q_c = 0 \tag{14}$$

$$M - M_c = 0 \tag{15}$$

where $(\circ)'$ denotes the derivatives with respect to s. Variables u and w are the components of a displacement vector and φ is the rotation of a cross-section at the reference axis. Variable ε and γ are extensional and shear strains, respectively, while κ represents the pseudocurvature (flexural deformation) of the beam reference axis. N, Q and M represent equilibrium generalized internal forces, while N_c, Q_c and M_c denote constitutive generalized internal forces. According to the given stress and strain state on the time interval i–1 and temperature on the time interval i, the mechanical strains $D^i = \varepsilon^i + z\kappa^i$ on the time interval i of any point in the curved beam can be calculated by the equation:

$$D^i = D^{i-1} + \Delta D^i, \tag{16}$$

where ΔD^i is the increment of the total strains (also known as the 'geometrical deformations') in the time interval i. Considering the principle of additivity of strains we end up with the strain increment, ΔD^i, as the sum of the strain increments due to temperature, ΔD^i_{th}, stress, ΔD^i_{σ}, creep, ΔD^i_{cr}, and transient strains of concrete, ΔD^i_{tr}:

$$\Delta D^i = \Delta D^i_{th} + \Delta D^i_{\sigma} + \Delta D^i_{cr} + \Delta D^i_{tr} \tag{17}$$

For a detailed description of each strain increment, see Hozjan (2011). The novelty of the presented mechanical model is the introduction of a new strain-based planar curved beam finite-element, which is based on the modified principle of virtual work. For a detailed description of the proposed model a reader is referred to Ružić (2013).

2 NUMERICAL EXAMPLE

We consider a curved RC beam exposed to the standard hydrocarbon fire curve as given in EC1 (EC1, 2004) simulating a severe tunnel fire conditions. The curved beam is combined of three radii *R1*, *R2* and *R3*. The geometrical, cross-sectional, material, reinforcement and loading data of the problem are shown in Fig. 1.

Other data used in the calculations are: density of concrete 2400 kg/m^3, density of cement per unit volume of concrete 300 kg/m^3, initial temperature 20 ^{o}C, initial pore pressure 0.1 MPa, initial water vapour content per unit volume of gaseous mixture 0.013 kg/m^3, boundary water vapour content per unit volume of gaseous mixture 0.104 kg/m^3, initial porosity of concrete

0.15, initial permeability of concrete $1 \cdot 10^{-16}$ and saturation water content at room temperature 100 kg/m³.

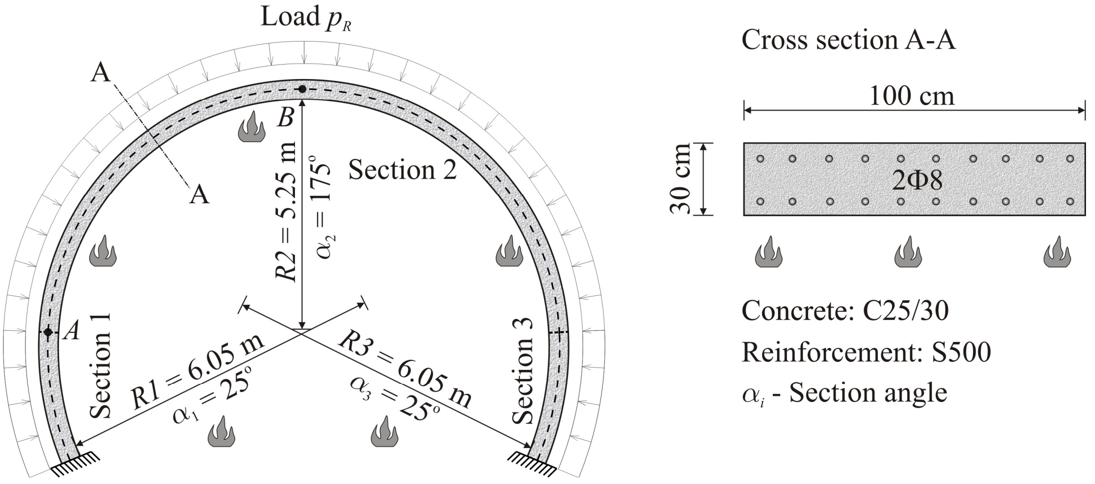
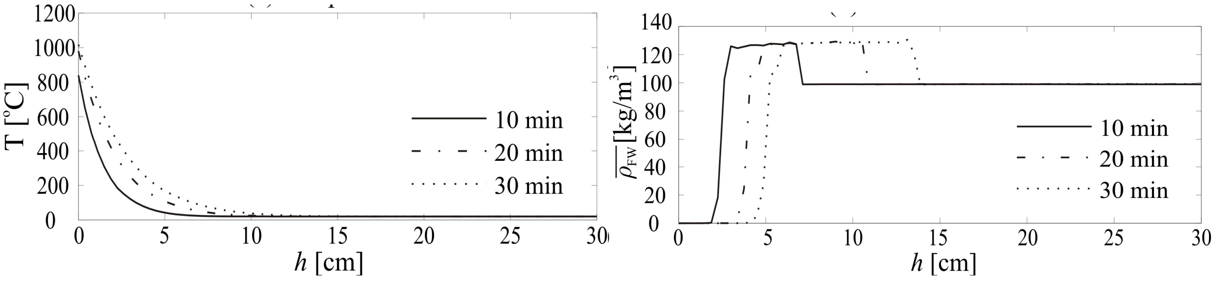


Fig. 1 Geometrical and material properties of clamped curved RC beam

In the hygro-thermal analysis, that we present first, a one dimensional numerical model for a characteristic cross-section of the beam is employed and a mesh consisting of 80 4-noded isoparametric finite elements is used. Furthermore, the time interval $\Delta t = 0.5\,\mathrm{s}$ is chosen (Hozjan et al., 2011).

The distribution of temperature over the cross-section of the beam at 10, 20 and 30 min is presented in Fig. 2a). As expected we can notice slow heating within the concrete due to its high heat capacity. For instance after 30 minutes of fire, temperature on the boundary is equal to around 1000° C, while the temperature in the point of rebar position being 5cm from the exposed surface is equal to only around 200° C. This means rebars maintain their initial strength also after 30 minutes of fire exposure.

Fig. 2b) shows the distributions of free water content over the cross-section. As observed from the figure we can notice that captured moisture inside the concrete beam follows the rise of temperature to change partly into vapour while being driven by the water pressure gradient towards the inward of the concrete beam. This driven flow of moisture can be observed in form of increased water front where values of free water content exceed initial values, while region right to the free water front is humid as initially. Region of increased water front mainly occurs due to the condensation of water vapour and by dehydration of chemically bonded water which is released in form of free water.



(a) (b)

Fig. 2 Distribution of (a) temperature and (b) free water content over the height h of cross-section at 10, 20 and 30 min

Once the temperature variation in time and space inside a curved RC beam during fire has been obtained, we start with the third phase of fire analysis. In what follows, we first present the convergence tests and then the parametric studies.

We investigated the rate of convergence of the proposed numerical method. Tests were performed for hinged and clamped curved beam with uniform load of $p_R = 270\text{kN/m}$. In Tab. 1 results for mid span displacement component w_B (point B) of clamped beam with respect to different ratio between the number of finite elements N_{el} and the degree of the interpolation polynomial N_{int} are presented. We can notice a good convergence.

Tab. 1 Rate of convergence of clamped curved RC beam

N_{el} / N_{int}	8 / 4	16 / 4	32 / 4	64 / 4	128 / 9
w_B (cm)	-2.36	-1.91	-2.12	-2.15	-2.14
Rel. er.(%)	10.048	10.866	1.230	0.181	0.000

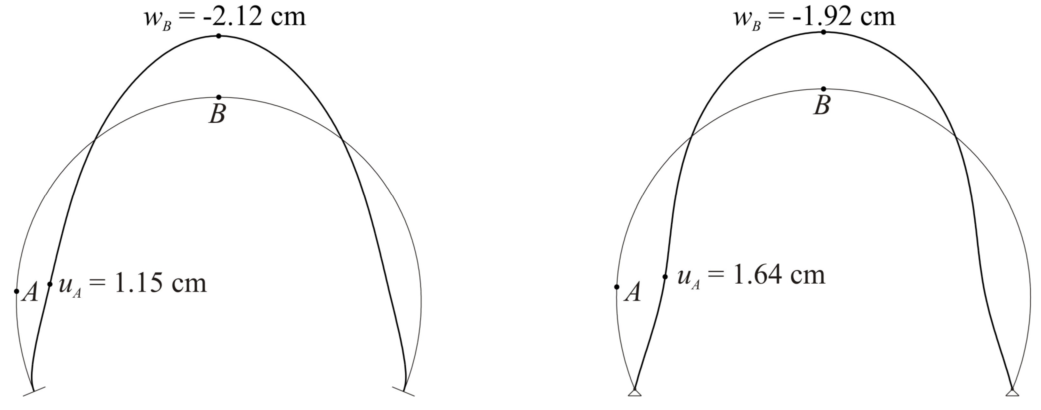
In addition we also investigated the rate of convergence for critical time and horizontal displacement component of point A (see Fig. 1.) which are not presented in the paper. The results were nearly the same as in the previous convergence test. Based on these results we decided to employ 32 finite elements with interpolation polynomial of the 4th order for parametric study.

Tab. 2 Critical time, displacement components of points A and B for clamped curved RC beam

Load p_R (kN/m)	N_{el}	t_{cr} [min]	w_B [min]	u_B [cm]
135	32	20.75	-2.45	1.10
270	32	35.97	-2.12	1.15
540	32	38.89	-1.83	1.44
810	32	17.04	-1.60	1.53

Tab. 3 Critical time, displacement components of points A and B for hinged curved RC beam

Load p_R (kN/m)	N_{el}	t_{cr} [min]	w_B [min]	u_A [cm]
135	32	10.58	-3.59	2.34
270	32	5.54	-1.92	1.64
540	32	9.87	-6.98	5.37



(a) t_{cr} = 35.97 min, p_{g} = 270 kN/m (b) t_{cr} = 5.54 min, p_{g} = 270 kN/m

Fig. 3 75 times scaled deformed shape of (a) clamped and (b) hinged curved beam at critical time

In the following parametric study different load levels p_R are considered for the observed curved RC beam accounting for hinged and clamped boundary conditions. Results are presented in Tab. 2 and 3 as well as in Fig. 3a) and 3b).

The presented parametric study indicates that load level and boundary conditions as well as their coupled interaction significantly affect the stability of the curved beam exposed to thermal loading. Deformed shape of the curved beam due to load and boundary conditions can cause instability as the unsupported lengths of the beam sectors change in the process of deforming.

3 ACKNOWLEDGMENT

The work of D. Ružić was partly financially supported by the European Union, European Social Fund. The support is gratefully acknowledged.

4 SUMMARY

We presented a new numerical model for the fire analysis of planar curved RC beams accounting for kinematical and material non-linearity. The temperature field in concrete was determined with a coupled model of slow transient phenomena involving heat and mass transport and pore pressure increase in concrete. Furthermore, the strain-based non-linear beam finite-element was involved in mechanical analysis. In the final numerical case the presented beam formulation has been found appropriate for the thermo-mechanical analysis of curved RC beams.

REFERENCES

Bažant Z.P., Kaplan M.F., Concrete at high temperatures: material properties and mathematical models. Longman, Harlow, 1996.

Bradford M.A., Buckling of circular steel arches subjected to thermal loading. Weld World, 50:349–9, 2006.

Davie C.T., Pearce C.J., Bićanić N., Coupled heat and moisture transport in concrete at elevated temperatures - Effects of capillary pressure and adsorbed water. Numerical Heat Transfer, Part A: Applications 49:733 – 763, 2006.

Heidarpour A., Pham T.H., Bradford M.A., Nonlinear thermoelastic analysis of composite steel-concrete arches including partial interaction and elevated temperature loading. Eng Struct, 32:3248–3527, 2010.

Hozjan T., Saje M., Srpčič S., Planinc I., Fire analysis of steel-concrete composite beam with interlayer slip. Comput Struct, 89(1-2):189–200, 2011.

Kolšek J., Fire analysis of two-layered composite structures, University of Ljubljana, Faculty of Civil and Geodetic Engineering, Doctoral thesis (in Slovene), 2013.

Reissner E., On one-dimensional finite-strain beam theory: The plane problem. Journal of Applied Mathematics and Physics (ZAMP), 23:795–804, 1972.

Ružić D., Fire analysis of partially delaminated curved reinforced concrete beam structures, Unpublished manuscript (in Slovene), University of Ljubljana, Faculty of Civil and Geodetic Engineering, 2013.

SIST EN 1991-1-2, Eurocode 1: Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire, 2004.

SIST EN 1992-1-2, Eurocode 2: Design of concrete structures – Part 1-2: General rules – Structural fire design, 2005.