NUMERICAL ANALYSIS OF TIMBER BEAM EXPOSED TO FIRE

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Abstract

In this paper the advanced calculation method to determine the response of timber beams during fire is presented. In the first phase of the advanced calculation method the development of temperature across the cross-section of the beam and the char dept are determined. The second phase consists of the mechanical analysis of timber beam. Char dept for one dimensional charring is compared to the empirical values from the literature. In addition, fire resistance calculated with advanced calculation model is compared with the fire resistance calculated with reduced cross-sectional method proposed by Eurocode (EN 1995-1-2 (2005)).

Keywords: heat transfer, advanced calculation method, reduced cross-sectional method, fire resistance, timber beam

INTRODUCTION

Fire safety of timber structures is strongly dependent on fire safety of timber elements that compose it. Timber beams represent one of such elements. From the view of fire safety timber structures are relatively safe in comparison to other types of structures. This is mainly due to relatively low thermal conductivity of timber and char. However, the temperature increase is deteriorating the load bearing capacity of timber elements. The temperature increase results in a reduction of mechanical properties of timber in addition, timber is subjected to thermal degradation or so-called pyrolysis. Pyrolysis is a complex phenomenon which starts at temperatures about 200-300°C and represents the combined effect of various chemical processes with the process of heat and moisture transfer. The results of pyrolysis are the formation of char, various gases, acids and resins. As a results the material properties of timber change. Char layer has both a positive and negative impact. It has a relatively low thermal conductivity which contributes to a slower warming in the core of the cross section of the timber element. On the other hand, the char layer has almost negligible strength characteristics and thus does not contribute to the load bearing capacity of the timber elements.

Fire safety of a timber beam can be proved in several ways. Standard Eurocode (EN 1995-1-2 (2005)) proposes simplified rules for analysis of structural members and components and advanced calculation methods. Simplified rules are primarily intended for the approximate evaluation of fire resistance of timber elements, e.g. beams or columns. The reduced cross-sectional method represents one of such simplified procedures. Fire resistance is calculated based on the effective cross-section, which is determined by reducing the initial cross-section by the effective charring dept where the design charring rate depends on standard fire curve ISO 834 (1999). The advanced calculation method allows us to analyse the behaviour of the structure and its part during the fire. The analysis is divided in tree separated phases. In the first phase the development of temperatures with time in the fire room is determined. The standard fire curve ISO 834 (1999) is considered. In the second phase the temperature state of timber beam is analysed, taking into account the charring of timber. Here the coupled problem of heat and moisture transfer is not modelled but the moisture content is indirectly considered with higher specific heat in the temperature range between 100 and 120°C (EN 1995-1-2 (2005)). Based on the temperature state of timber beam the mechanical analysis is determined

in the third phase. Additive principle is adopted where the increment of the total geometric strain is divided on the increment of the mechanical and thermal strain.
The aim of this paper is to provide comparisons between both procedures which are proposed by Eurocode.

1 THERMAL ANALYSIS

1.1 Heat transfer

To determine time dependent temperature in cross section of timber element all three ways of heat transfer are considered: convection, radiation and conduction. Heat conduction over cross-section is described with Fourier partial differential equation:

$$\left(k_{\mathrm{ij}} \cdot \frac{\partial T}{\partial x_{\mathrm{j}}}\right) \frac{\partial}{\partial x_{\mathrm{i}}} + Q - \rho \cdot c \cdot \frac{\partial T}{\partial t} = 0. \tag{1}$$

Heat transfer through outer surface of the beam due to convection and radiation is considered with appropriate boundary conditions. These are:

$$S_{\mathrm{q}} : q^{\mathrm{s}} = -k_{\mathrm{ij}} \cdot \frac{\partial T}{\partial x_{\mathrm{j}}} n_{\mathrm{i}}, \qquad S_{\mathrm{T}} : T^{\mathrm{s}} = T, \qquad S : T(t=0) = T_0. \tag{2}$$

In equations (1) and (2) S denotes cross section of beam, S_q is a part of cross-section where specific surface heat flux q^s is prescribed, S_T represents a part of cross-section where temperature T_S is prescribed, t is time, k_{ij} constitutes the symmetric thermal conductivity tensor, $\partial T/\partial x_{\mathrm{j}}$ is the partial derivate of temperature over coordinate x_j, n_i is a component of the unit vector perpendicular to the cross-section, Q is the internal heat source, ρ the density of material, c specific heat and T_0 is the initial temperature at any point of the cross section. Specific surface heat flux consists of the share represented by the exchange of heat between the body and the surrounding area by convection (q_c), the share of the radiation (q_r) and from other sources (q_0).

$$q^{\mathrm{s}} = q_{\mathrm{c}} + q_{\mathrm{r}} + q_0. \tag{3}$$

Heat flux due to convection q_c depends on the temperature of gases in the vicinity of the fire exposed element T_s, the surface temperature of the element T and coefficient of heat transfer by convection α_c. Heat flux due to radiation q_r depends on the emissivity of the surface of the element ε_m, Stefan-Boltzman constant σ and the difference between the effective radiative fire temperature T_r and the surface temperature of the element T.

$$q_{\mathrm{c}} = \alpha_{\mathrm{c}} \cdot (T_{\mathrm{s}} - T), \qquad q_{\mathrm{r}} = \varepsilon_{\mathrm{m}} \cdot \sigma \cdot \left(T_{\mathrm{r}}^4 - T^4\right). \tag{4}$$

System of equations (1)-(4) is solved numerically by the finite element method in the *Matlab* environment.

1.2 Material properties of timber and the char layer at elevated temperatures

Temperature dependence of the specific heat of timber, density ratio for softwood and thermal conductivity for timber in accordance with standard EN 1995-1-2 (2005) are shown in Fig. 1. With the increase of the specific heat of timber in temperature range between 100 and 120°C the indirect impact of evaporation of water on the development of the temperature in timber is modelled. Thermal conductivity suggested by the EN 1995-1-2 (2005) takes into consideration increased heat transfer due to shrinkage crack at temperatures above 500°C.

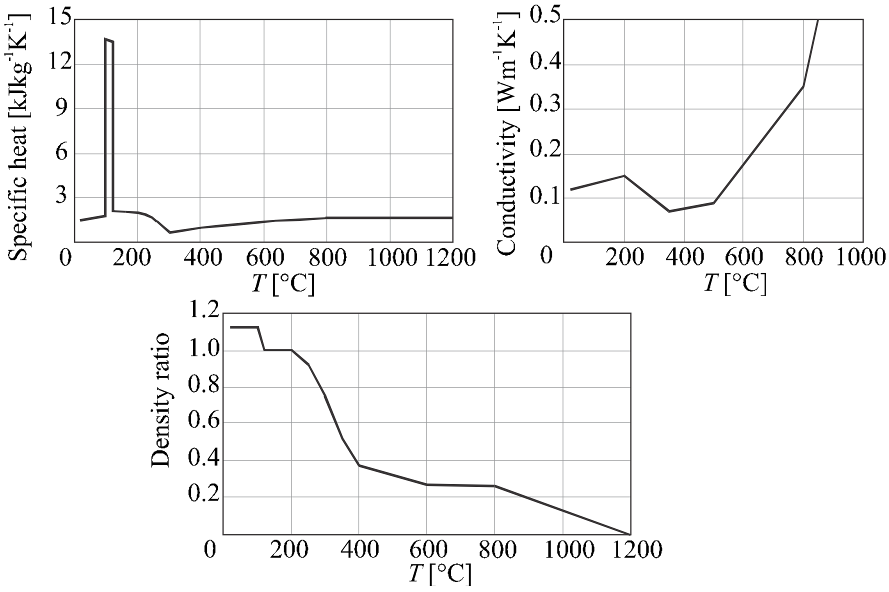

Fig. 1 Temperature-specific heat relationship for timber and char, temperature dependent density ratio for softwood (for initial moisture content of 12%) and temperature dependent thermal conductivity for timber and the char layer (EN 1995-1-2 (2005))

2 MECHANICAL ANALYSIS

The presented finite element formulation is based on Reissner's kinematically exact model of beam where large membrane and flexural deformations are allowed (Reissner, 1972). The effect of the shear strain is neglected. The geometric extensional strain is a function of extensional strain of centroidal axis ε and its pseudocurvature κ. The Bernoulli hypothesis is considered and the geometric extensional strain over the beam cross section can be written:

$$D(x,z)=\varepsilon(x)+z\kappa(x). \tag{5}$$

Basic equations for beam are presented by kinematic, equilibrium and constitutive equations:

$$\begin{aligned}
&1+u'-(1+\varepsilon)\cos\varphi=0, && (N\cos\varphi+Q\sin\varphi)'+p_x=0, && N_c=\int_A \sigma(D_m,T)\,dA,\\
&w'-(1+\varepsilon)\sin\varphi=0, && (-N\cos\varphi+Q\sin\varphi)'+p_z=0, && \\
&\varphi'-\kappa=0, && M'-(1+\varepsilon)Q+m_y=0, && M_c=\int_A z\sigma(D_m,T)\,dA.
\end{aligned} \tag{6}$$

The prime $(\ )'$ denotes the derivative with the respect to x, u and w are displacements of the centroidal axis in the x and z direction, φ is the rotation about y-axis. N, Q, M are equilibrium generalised internal forces, p_x, p_z and m_y denotes conservative distributed loads of the element. Constitutive internal forces N_c and M_c depend on a chosen material model which is defined by the relationship between the longitudinal normal stress $\sigma(D_m,T)$ and mechanical extensional stress D_m, of a longitudinal fibre at elevated temperature. The geometric extensional strain is determined using incremental equation:

$$D^j=D^{j-1}+\Delta D^j, \tag{7}$$

where D^j and D^{j-1} denotes the total geometric strains in the time intervals j and j-1, ΔD^j is the increment of the total geometric strain in the time interval j an it is assumed to be the sum of mechanical extensional strain increment ΔD^j_m and thermal strain increment ΔD^j_T.

$$\Delta D^j=\Delta D^j_m+\Delta D^j_T. \tag{8}$$

The final system of equations for finite element method is written based on modified principle of virtual work where quantities ε and κ are interpolated over finite element by Lagrangian polynomials. A more detailed description of the finite element formulation is presented in Bratina et al (2003).

2.1 Mechanical properties of timber at elevated temperatures

Mechanical properties for strength and modulus of elasticity parallel to the grain of softwood are considered in accordance with EN 1995-1-2 (2005). Reduction factors are different for timber fibre in tension or compression. It is considered that the char layer doesn't have any strength. The char occurs at a temperature around 300°C therefore the reduction factors above this temperature are equal to zero.

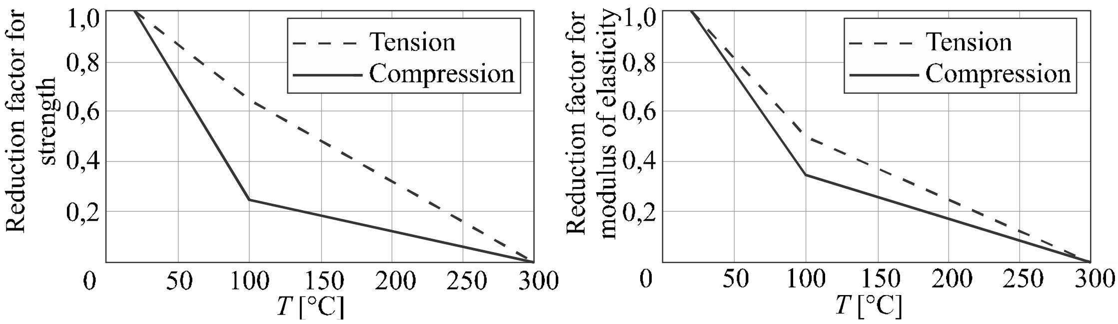

Fig. 2 Reduction factor for strength and modulus of elasticity parallel to the grain of softwood (EN 1995-1-2 (2005))

The normal stress σ and mechanical extensional strain D of a longitudinal fiber are connected thru linear relationship in tension and bi-linear relationship in compression (Fig. 3).

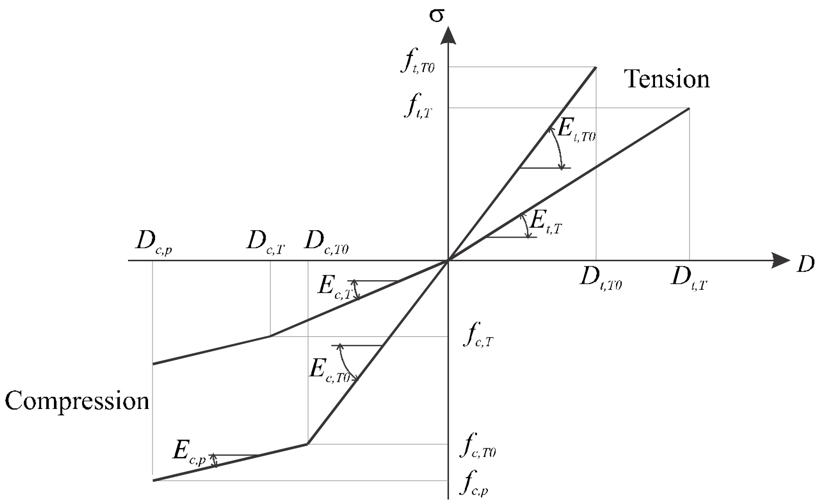

Fig. 3 Stress-strain relationship for timber at ambient and elevated temperature

In Fig. 3, $D_{i,j}$, $E_{i,j}$ *and* $f_{i,j}$ ($i = c, t$; $j = T, T_0$) are the limit elastic strains, the Young's modulus and the limit elastic stresses for timber in compression (c) and tension (t) at ambient (T_0) and elevated temperatures (T). $E_{i,j}$ *and* $f_{i,j}$ are determined according to Fig. 2 and $D_{i,j}$ is determined from their relationship. The limit plastic stress $f_{c,p}$, is defined by Pischl (1980). Symbols $E_{c,p}$ and $D_{c,p}$ denotes plastic hardening modulus and limit plastic strain for timber in compression.

3 CASE STUDIES

3.1 One dimensional charring of timber beams

In this example comparison is made between present model of charring and different models of one-dimensional charring (EN 1995-1-2, 2005; Schnabl, 2007). In the present model we consider that the charring occurs at a temperature of 300°C. Specific heat, conductivity and density are taken into account in accordance with EN 1995-1-2(2005) and are shown on Fig. 1. Standard ISO fire curve, coefficient of heat transfer by convection $\alpha_c = 25$ W/m^{2}K and the emissivity of the surface $\varepsilon_m = 0.8$ are considered in thermal analysis. The initial moisture content w is 0.12 and the initial density is $\rho = 380$ kg/m^3. The cross section considered for the analysis is $b/h = 5/30$ cm.

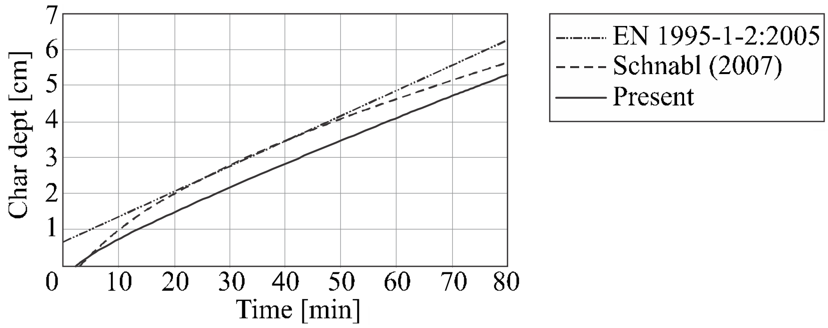

Fig. 4 Comparison of different one-dimensional charring models

The start of charring in the present model occurs at the approximate time of 3 min which is the same as by model proposed by Schnabl. In the model proposed by Eurocode, the charring begins simultaneously with the start of fire, which does not represent the actual state. Char depth increases almost linearly with time and it fits well with the values proposed by Schnabl in the beginning and at the end of the simulation, while bigger discrepancy can be observed for time between 20 and 70 minutes. In the present model the char depth at the time of 80 minutes is 5.34 cm, 5.56 cm for model presented by Schnabl and 6.1 cm in case of Eurocodes.

3.2 Fire resistance of timber beam

Subject of our analysis is simply supported timber beam with span of 3 m. It is loaded with uniform load of 5 kN/m. The cross section of the beam is $b/h = 10/20$ cm. The strength class of timber is C30. The characteristic bending strength $f_{m,k}$ and Young's modulus for selected timber at ambient temperature are 3 kN/cm^2 and 1200 kN/cm^2.

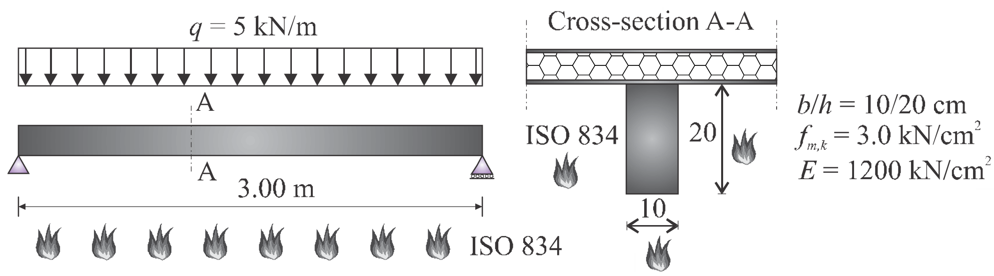

Fig. 5 Data for simply-supported timber beam

3.2.1 Simplified rules - Reduced cross-sectional method

Method is well described in EN 1995-1-2(2005), therefore we present only the parameters needed for the calculation of the fire resistance of timber beam. The design notional charring rate under standard fire exposure, β_n is 0.7 mm/min, depth of layer with assumed zero strength and stiffness, d_0 is 7 mm, coefficient k_0 is 1, the modification factor for fire $k_{mod,fi}$ is 1 and the design strength in fire $f_{d,fi}$ is 3 kN/cm^2. Failure time of timber beam determined by reduced cross-sectional method is 33.5 min.

3.2.2 Advanced calculation method

For the thermal analysis, the specific heat, density and thermal conductivity are taken based on the Fig. 1, where the initial density is ρ = 380 kg/m^3. Standard ISO 834 curve, the emissivity of the surface of the element ε_m = 0.8 and the coefficient of heat transfer by convection α_c = 25 W/m^{2}K are taken into account. In mechanical analysis, the parameters for stress-strain relationship for timber are as follows: $E_{t,T0} = E_{c,T0}$ = 1200 kN/cm^2, $f_{t,T0} = f_{c,T0}$ = 3 kN/cm^2, $D_{t,T0} = D_{c,T0}$ = 0.0025, $D_{c,p}$ = 0.0065, $E_{c,p}$ = 250 kN/cm^2 and $f_{c,p}$ = 4 kN/cm^2. Thermal strain is considered linear with a coefficient of linear thermal expansion $\alpha_T = 5\cdot10^{-6}$ m/m°C.

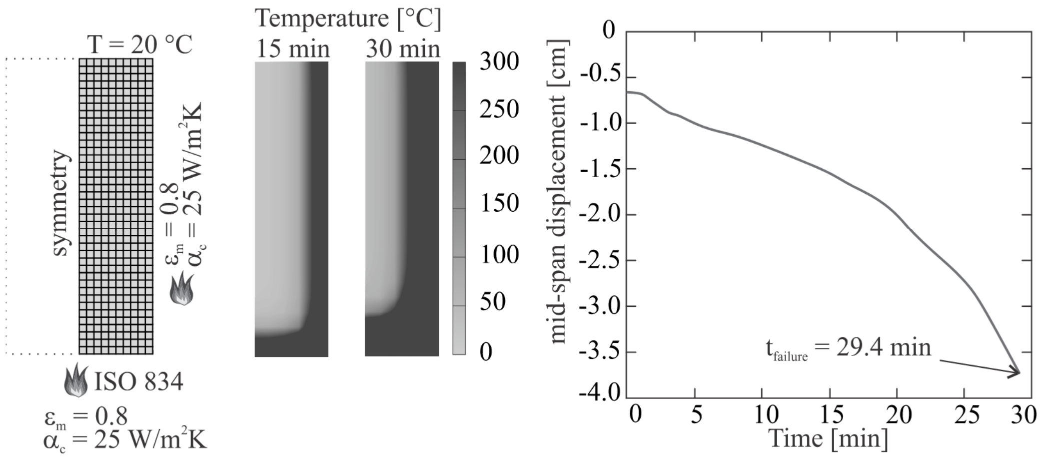

Fig. 6 Finite element mesh, temperature distribution in the cross-section and the time-displacement curve of the analyzed timber beam

The initial mid-span displacement is 0.66 cm. To the time of 20 minutes the mid-span displacement increases slower, after that time the increase is faster. The failure time of the timber beam occurs at 29.4 minutes where the mid-span displacement is 3.74 cm. Failure occurs when the longitudinal fibres are fully exploited and cannot provide any more to the bearing capacity of the cross-section of the beam. Compared to the reduced cross-sectional method the failure time is smaller but remains in the same rank. The difference in failure time for both methods is 4.1 minutes which is 12.2 %. From the view of fire-safety design the advanced calculation method for this case gives more conservative result.

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